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Elastic Stiffness and the Economics of Fibre-Reinforced Plastics

J. SCANLAN
Imperial Chemical Industries Ltd, Petrochemical and Polymer Laboratory, The Heath, Runcorn, Cheshire, U K

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Present-day knowledge of the relationships between some mechanical properties of a fibre-reinforced composite and those of its components is briefly reviewed. On the basis of this knowledge estimates can be made of the elastic stiffness likely to be achieved for polymeric resins containing the new high-performance fibrous reinforcements. Using these estimates, the cost-effectiveness of such composites in meeting several different rigidity criteria has been investigated.

1. Introduction

The advance of engineering raises ever more stringent requirements in material properties. In the hope of meeting these needs, attention has been focused on the elements in the centre of the first periods of the periodic table, and on the compounds between them. The high covalency of these elements gives to such materials high elastic moduli, high melting and boiling points and, theoretically at least, high mechanical strengths, in spite of their relatively low densities. Their extremely brittle nature and sensitivity to minute surface or internal imperfections mean that these materials are not useful as structural materials in massive form, but must be used in the form of fine fibres or filaments dispersed in a suitable matrix. Properties of some of these materials are given in table I together with those of glass fibre and steel wire; the properties are most outstanding when the materials are in the form of "whiskers" (long, fine, needle crystals), but some have also been prepared as continuous filaments, and give excellent moduli in this form, although the strength is then considerably lower.

In certain applications such as aircraft, and especially, spacecraft, weight-saving is of great economic value, and the high strength and stiffness/weight ratios which can be anticipated for fibre-reinforced composites are likely to make them acceptable, even if their cost is high. Large-scale production of the high-performance reinforcements would be expected to reduce

TABLE I Properties of fibres.

Material	Specific gravity	Tensile modulus (10^6 lb/in.^{2}*)	Tensile strength (10^6 lb/in.^{2}*)	Specific modulus E/(S.G.) (10^6 lb/in.^{2}*)	Specific strength T.S./(S.G.) (10^6 lb/in.^{2}*)
Whiskers					
Carbon [1]	2.2	98	2.8	45.0	1.3
Alumina [1]	4.0	76	2.2	19.0	0.55
Silicon carbide [1]	3.2	100	3.0	31.0	0.94
Silicon nitride [1]	3.1	55	2.0	18.0	0.64
Continuous filaments					
Steel [1]	7.8	30	0.6	3.8	0.08
E glass [2]	2.55	10.5	0.5	4.1	0.2
Carbon [3]	2.0	60	0.3	30.0	0.15
Boron [3]	2.5	60	0.37	24.0	0.15

*1.0 lb/in.^{2} = 7.0 × 10^{-2} kg/cm^2

markedly their present high price, and it is the purpose of this paper to examine the possibility of composite materials containing such materials becoming competitive with conventional materials such as metals, wood, and unreinforced plastics, in situations where weight-saving, although possibly useful, cannot justify a premium.

2. The Mechanical Properties of Fibre-Reinforced Plastics

Published analyses [4-6] of the elastic moduli and tensile strengths of composites containing continuous filaments have been based essentially on considering the reinforcement and the matrix to act in parallel and lead to the simple expressions

$$E_c = (1 - \phi)\, E_m + \phi \eta E_r \qquad (1)$$
$$G_c = (1 - \phi)\, G_m + \phi \eta' E_r \qquad (2)$$
$$\sigma^u{}_c = (1 - \phi)\, \sigma'_m + \phi \eta \sigma^u{}_r \qquad (3)$$

where E_c, E_m, and E_r are Young's moduli for the composite, matrix, and reinforcement; G_c and G_m are the shear moduli of the composite and matrix; $\sigma^u{}_c$ and $\sigma^u{}_r$ are the breaking stresses of the composite and reinforcement, while σ'_m is the stress in the matrix at an extension equal to the breaking extension of the reinforcement; ϕ is the volume fraction of the reinforcement in the composite; η and η' can be called the reinforcement efficiency factors [6].

For fibres randomly aligned in a plane, the appropriate values of η and η' are $\frac{1}{3}$ and $\frac{1}{8}$ respectively. If the fibres can be used in the most effective way, the appropriate values are $\eta = 1$ with all fibres parallel to the direction of stress in simple elongation or compression, and $\eta' = \frac{1}{4}$ with all fibres at 45° to the shear direction in simple shear.

Justification for use of the equations 1 to 3, derived from what is obviously an over simple and approximate theory, can be obtained from the experimental data on composite properties. Composites containing continuous filaments aligned in the tensile-strain direction have been investigated by McDanels, Jech, and Weeton [7], and by Kelly and Tyson [8], using in both cases tungsten wires in a copper matrix. Kelly and Tyson have also investigated discontinuous parallel fibres in some detail, but in this paper it will always be assumed that the fibres are sufficiently long for end effects to be insignificant.

Systematic investigations of composites with other reinforcement orientations are rather lacking. Krenchel [6] has investigated glass-fibre-reinforced thermosetting resins, with the reinforcement in a number of forms. Table II is adapted from his results. Equations 1 and 2 have been used to calculate, from the properties of the composites and of the resin matrices, values for the modulus and strength of the glass.

TABLE II Krenchel's results on glass-fibre-reinforced thermosetting resins.

Reinforcement	Resin	ϕ	E_c (kg/cm² × 10^{-5})	$\sigma^u{}_c$ (kg/cm² × 10^{-3})	$e^b{}_c$ (%)	E_m (kg/cm² × 10^{-5})	η	E_r (kg/cm² × 10^{-5})	$\sigma^u{}_r$ (kg/cm² × 10^{-3})	Notes
Strands					1.24-1.75			4.5-6.4	2.5-9.8	
Strands from mat									12.6	
Roving	Epoxy	0.529	4.12	8.58	2.1	0.19	0.98	7.8	16.2	1
Yarn	Polyester	0.232	1.75	3.37	2.1	0.21	1.0	6.9	13.1	
Diamond mat	Epoxy	0.558	3.64	2.35		0.11	0.98	6.6		2
Weave	Polyester	0.540	2.2	3.08	1.8	0.08	0.60	6.7	9.3	3
Weave	Polyester	{0.324	1.43	2.62	(1.8)	0.21	0.58	6.9	12.6}	4
		{0.322	1.56	2.65		0.39		7.0	11.7}	
Weave	Epoxy	{0.507	2.15	3.19	1.8	0.36	0.52	7.5	10.9}	5
		{0.488	1.95	2.82			0.48	7.6	10.6}	
Mat	Polyester	0.595	1.79	1.98	1.6	0.08	0.333	8.9	9.7	6
Mat	Polyester	0.105	0.47	0.64	1.6	0.21	0.333	8.0	9.7	
Mat	Epoxy	0.291	1.07	1.52	1.5	0.19	0.333	9.7	13.6	
Fabric	Polyester	0.172	0.66	0.72	1.5	0.21	0.333	8.5	8.0	

1. Slight twist given during formation of composite reduces reinforcement efficiency from unity.
2. Mat consisted of two sets of strands inclined at 12°; failure occurred by shear in this case.
3. For woven cloths the reinforcement efficiency is the fraction of the reinforcement in the direction of the applied stress.
4. Two different curing cycles.
5. Tests in two perpendicular directions.
6. For mats and non-woven fabric reinforcing filaments assumed randomly oriented in plane.

The calculated values of the glass modulus are reasonably consistent and in good agreement with the expected value (7.4×10^5 kg/cm^2) except that the mats and fabric give rather high values; the calculated strengths are more scattered, and much lower than the strength of fresh glass fibres.

3. The Economics of Fibre-Reinforced Plastics

A new structural material has to struggle for acceptance against the materials currently in use, and were these not both generally (although never completely) satisfactory and cheap, technology could not have reached its present level. It is necessary, therefore, to make a preliminary economic comparison between plastic containing the new reinforcement, and some common structural materials. The costs and other properties assumed for these materials and for the plastic matrix and the reinforcement are given in table III.

Different applications require, of course, different design criteria, and these criteria will often be complex, with more than one requirement to be satisfied; for example lower limits might be set simultaneously on strength, rigidity, resistance to corrosion, fire resistance and so on. Several simple elastic criteria will be considered below. In most applications a criterion of this type will be crucial and other requirements can be considered as subsidiary.

Applications in which the chief requirement is for a panel with a certain resistance to flexure are considered first. Examples would be cladding panels for buildings and car-body parts not involved in the main body/chassis structure such as boot (trunk) lids, bonnets (hoods) and doors.

The thickness of a sheet or depth of a beam needed to meet a design requirement for a maximum deflection under a given load is inversely proportional to the cube root of the Young's modulus so that the cost of a panel is proportional to

$$Cd/E^{\frac{1}{3}}$$

where C is the cost of the material of the panel per unit weight; d is the density of the material; and E is its modulus.

Fig. 1 shows the effect of adding reinforcement on the cost of a plastic panel for several different reinforcement costs, the properties of resin and reinforcement being those given in table III.

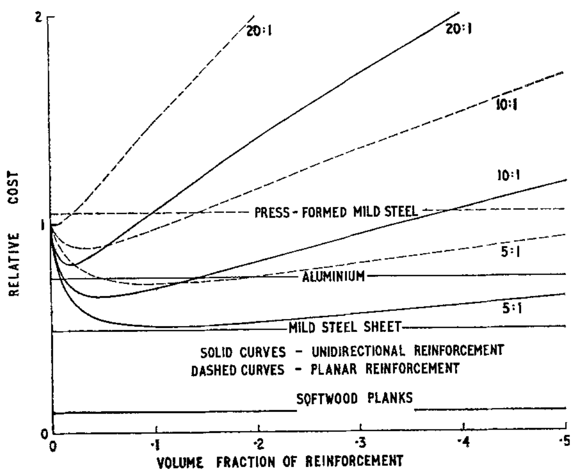

Figure 1 Relative costs of reinforced and unreinforced panels of specified flexural rigidity. The figures against the curves give the ratio of volume costs of reinforcement and matrix.

From equation 1 the relative costs of a reinforced and unreinforced panel are given by

$$\frac{1 - \phi + c\phi}{(1 - \phi + e\phi)^{\frac{1}{3}}}$$

TABLE III Costs and properties used in comparisons.

Material	Cost (d/lb†)	Spec. grav.	E (lb/in.^{2}‡)	Cost relative to cost of resin	Relative cost of panel on $E^{\frac{1}{3}}$ basis	on E basis
Resin	24	1.0	3×10^5	1.0	1.0	1.0
„ formed	48					
Fibre reinforc.	Variable		6×10^7			
Softwood sawn	7	0.5	1×10^6	0.29	0.10	0.044
Steel sheet	7	7.8	3×10^7	0.29	0.49	0.023
„ pressed	30			0.625*	1.05*	0.049*
Aluminium ingot	21	2.7	1×10^7	0.875	0.74	0.072

*Compared to formed cost of resin
†1.0 lb = 0.45 kg
‡1.0 lb/in.^{2} = 7.0×10^{-2} kg/cm^2

where c is the relative cost of reinforcement and resin matrix on a volume basis, and $e = \eta E_r/E_m$. Two cases are considered: a random distribution of fibre directions in a plane, and the most efficient use of the reinforcement in a beam, i.e. parallel fibres aligned along the beam length. Also shown as full horizontal lines are the relative costs of some competitive materials.

In this case the cost of the panels shows a definite minimum and the economy which can be achieved by incorporation of a fibrous reinforcement is very limited. With a volume cost of the fibre twenty times higher than that of the resin matrix, the planar reinforcement (which is likely to be the most important in practice) gives almost no saving in material cost even ignoring the mixing cost which will also be involved. To achieve even a 30% reduction in material cost under these conditions would require a volume cost of the reinforcement down to about five times that of the resin, corresponding, with our assumed cost of the resin of 2s/lb (1.0 lb = 0.45kg) and taking the density of high-modulus fibres as twice that of the resin, to about 5s/lb for the fibres – an extremely optimistic figure for future cost.

In this type of application, unreinforced plastics are most competitive with the other materials, their low moduli being a minimal disadvantage which is largely offset by their low density and consequent low cost per unit volume. It is thus not surprising that the cheapening effect of high-modulus filler is not large.

One factor which has not so far been considered but which is of prime importance is the cost of forming. One major advantage of thermoplastics is the ease with which they can be formed into complex shapes by such processes as injection moulding. Indeed were it not for this their uses would be very limited. It is very difficult to make valid comparisons of the cost of fabricated articles in different materials. Much depends upon the length of production run and similar factors. However, as an illustration, the cost of a press-formed mild steel panel as compared to that of a moulded thermoplastic panel of similar flexural rigidity is also shown in the figure. The fabrication costs are believed to be of the order of those actually achieved in industry and in both cases are well above calculated ideal figures. The competitive position of the plastic is considerably improved when forming costs are included. This is certainly a reflection of the true industrial situation.

Without much investigation it would not be possible to estimate accurately the effect of incorporation of reinforcement on fabrication cost. Easy moulding will certainly require a limit on the amount of reinforcement used and will thus favour dearer high-modulus fillers over cheaper ones of lower modulus. Filament-winding techniques will not impose the same restriction but can only produce simple shapes.

The most favourable assumption about moulding costs is that they depend only on the volume moulded, and in this case the curves for relative costs of the panels have the same shape, but correspond to higher reinforcement costs, for in this case the ratio of the volume costs of reinforced and unreinforced panels is not $(1 - \phi + c\phi)$ but

$$\frac{1 - \phi + f + c\phi}{1 + f} = 1 - \phi + \left(\frac{c + f}{1 + f}\right)\phi$$

where f is the ratio of fabrication to material cost for the unreinforced panel, giving the same form with $(c + f)/(1 + f)$ replacing c.

In other applications it is not appropriate to increase the depth of a beam to increase its stiffness as overall dimensions are settled by other design factors. With modern monocoque methods of car construction, for example, the body shell provides the bending and torsional stiffness required. Regarding such structures as hollow box girders with the wall thickness small compared to the overall depth, the resistance to bending is determined by Young's modulus × thickness, and the material cost of the structure in this case is proportional to Cd/E, and the relative costs of reinforced and unreinforced plastic panels are given by

$$\frac{1 - \phi + c\phi}{1 - \phi + e\phi}$$

In such a comparison involving Young's modulus directly, unreinforced plastics are at a great disadvantage against metals and even against wood. Reinforcement with a high-modulus fibre greatly improves the competitive position of plastics here (fig. 2) in distinction from the flexure case considered above. However, it seems very unlikely that the price of the filler will ever be sufficiently low for material cost to be within reach of those of the common metals for, ignoring resin cost altogether, a fibre such as has been considered, with a density of 2.5, used in a two-dimensional fashion, would have to have a

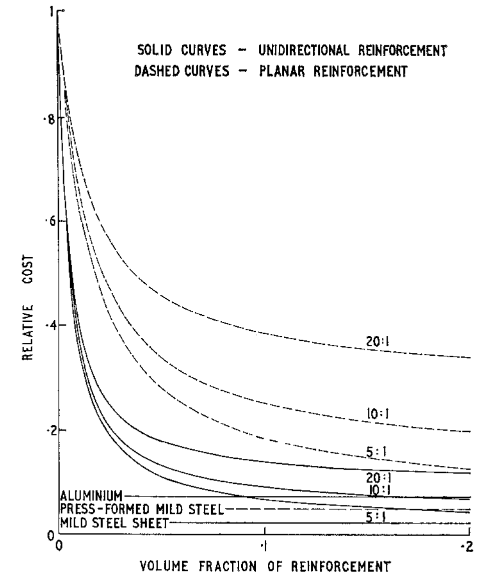

Figure 2 Relative costs of reinforced and unreinforced panels of specified resistance to extension.

specific cost less than twice that of mild steel to be competitive.

A similar picture is obtained in the shear case (fig. 3) as in the second of the two cases above.

Even if reasonably low prices can be attained for high-modulus fibres, large-scale use can only be hoped for if economies in production could be obtained with the composites to compensate for the high material cost.

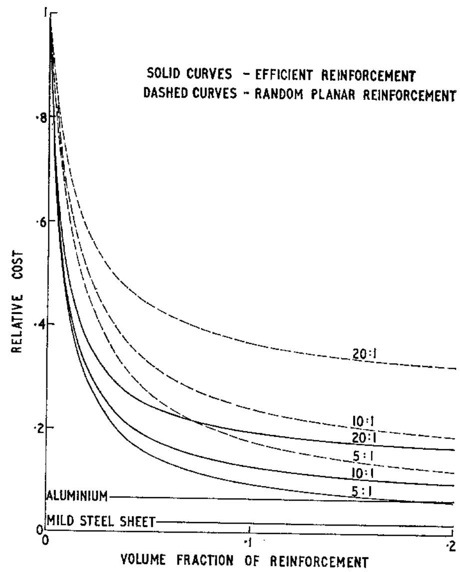

Figure 3 Relative costs of reinforced and unreinforced panels of specified shear resistance.

The situation with the high-modulus fibres could well be similar to that of glass-fibre-reinforced plastics, whose acceptance as engineering materials, although growing, has not been rapid. It is interesting that these materials have made a very significant penetration into small-boat construction, a field which illustrates some of the factors discussed. The principal requirement is for stiffness of the hull structure so that reinforcement of the resin used is very worthwhile, although the need to provide stiffness in the individual panels probably reduces the saving below that theoretically attainable. The decisive factor in replacement of the traditional wood, however, is cheaper fabrication of the rather complex shape required, while the small scale of production makes impossible the adoption of low-unit-cost but high-capital methods of metal forming. The resistance of the composites to corrosion and rotting represents a valuable bonus.

Conclusions

(i) Reinforcement of plastics with high-modulus fibres can be of little economic value in meeting needs for flexural stiffness.

(ii) In applications requiring extensional or shear stiffness it is very unlikely that material costs of composites will not be considerably higher than those for metals. Production economies will have to be realised if adoption of the composites as engineering materials is to be possible.

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